Price competition, cost and demand disruptions and coordination of a supply chain with one manufacturer and two competing retailers

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1 Introduction

■ Background:

    Generally, supply chain coordination management investigates a static coordination mechanism under a deterministic environment such as a known market demand and production cost. But the plan is often disrupted by some haphazard events. So it is important to know how the supply chain can be still coordinated under different disruptions.

■ Problem to deal with:

    How the supply chain which has 1 manufacture and 2 competing retailers can be coordinated under the production cost disruption incurred to the manufacturer.
2 Model

2.1 Static game with complete information based on Bertrand model.

- Price competition in static oligopoly model
  - Market is dominated by small number of sellers → We called 2 sellers situation duopoly.
  - The decisions of one seller are influenced by decisions of other sellers.
  - Sellers are neither the subjectives nor objectives of prices, but the seekers.

- Bertrand model
  - Sellers compete in price → Choice variable is price.
  - Method to get Nash Equilibrium:
    - Each sellers must behave optimally assuming that rival behaves optimally.
    - Mathematically, seller i's profit function is differentiable in \( p_i \), we drive each seller's first-order condition with respect to its own price for a given rival price and solve the system of equations.
2 Model

2.2 Basic model

- **Parameters**

  - $c_0$ --- unit cost for manufacture produce goods
  - $c_i$ --- unit cost for retailers add values
  - $\omega_i (i = 1, 2)$ --- unit wholesale price for retailers
  - $p_i$ --- retail price for retailers
  - $q_i$ --- demand of retailers
  - $a$ --- market scale
  - $d$ --- substitutability coefficient

- **Basic model**

  Linear market demand for retailer $i (j \neq i)$
  \[
  q_i(p_1, p_2) = a - p_i + dp_j, \quad 0 < d < 1, \quad (1)
  \]

  The total profit of the supply chain is
  \[
  \Pi(p_1, p_2) = \sum_{i=1, j \neq i}^{2} (p_i - c_i - c_0)(a - p_i + dp_j). \quad (2)
  \]

  Solving the first-order conditions $\frac{\partial \Pi}{\partial p_i} = 0, i = 1, 2$, we know that the optimal solution of optimal quantity of retailer $i$ in the centralized supply chain is
  \[
  p_i^* = \frac{a}{2(1-d)} + \frac{1}{2}(c_0 + c_i) > 0. \quad (3)
  \]

  According to (1)(3), we got optimal quantity of retailer $i$ in the centralized supply chain is
  \[
  q_i^* = \frac{1}{2}[a - c_i + d c_j - (1-d)c_0] > 0
  \]
2.3 Supply chain

- Centralized supply chain
  The supply chain is managed by a central planner who is able to control all decisions. The set of actions that can optimize the supply chain’s performance is called the centralized optimal solution.

- Decentralized supply chain
  Members within a supply chain to be independent organizations that aim to maximize their own objectives. The behavior of a decentralized supply chain can be characterized using the Nash equilibrium concept.

- Coordinated supply chain
  A supply chain is coordinated if all supply chain members adopt the actions that optimize the entire system’s performance.
2 Model

2.4 Centralized solution with cost disruption

- **Parameters**
  - \( \tilde{c}_0 \) --- production cost with disruption, \( \tilde{c}_0 = c_0 + \Delta c > 0 \)
  - \( \tilde{q}_i \) --- demand of retailer \( i \) with production cost disruption
  - \( c_u \geq 0, c_s \geq 0 \) --- penalty cost for deviation production

- **Model of the centralized supply chain with cost disruption**
  
  \[
  \Pi(\tilde{p}_1, \tilde{p}_2, \Delta c) = \sum_{i=1, j \neq i}^2 (\tilde{p}_i - c_0 - \Delta c - c_i)(\tilde{p}_i - \tilde{q}_i) - c_u [2a - (1 - d) \tilde{p}_1 - q_1^* - q_2^*] - c_s [q_1^* + q_2^* - 2a + (1 - d) \tilde{p}_1 + \tilde{p}_2]^+ (4)
  \]

  - total profit of the supply chain without the deviation penalty
  - total penalty cost incurred by the increased production.
  - total penalty cost incurred by the decreased production.

  \[
  \frac{\partial \Pi}{\partial \tilde{p}_i} = 0 \quad \text{we get optimal solution of (4),}
  \]

  \[
  \tilde{p}_i^* = \begin{cases} 
  p_i^* + \frac{1}{2}(\Delta c + c_u) & \text{if } \Delta c \leq c_u, \\
  p_i^* & \text{if } -c_u < \Delta c < c_s, \\
  p_i^* + \frac{1}{2}(\Delta c - c_s) & \text{if } \Delta c \geq c_s,
  \end{cases} \quad i = 1, 2.
  \]

  \[
  \tilde{q}_i^* = \begin{cases} 
  q_i^* - \frac{1}{2}(1 - d)(\Delta c + c_u) & \text{if } \Delta c \leq c_u, \\
  q_i^* & \text{if } -c_u < \Delta c < c_s, \\
  q_i^* - \frac{1}{2}(1 - d)(\Delta c - c_s) & \text{if } \Delta c \geq c_s, \\
  \end{cases} \quad i = 1, 2.
  \]

  Inserting the optimal retail prices into (1), we find that the optimal order quantities are...
2 Model

2.4 Centralized solution with cost disruption

- Theorem 1

The optimal solution above is

\[
\tilde{p}_i^* = \begin{cases} 
  p_i^* + \frac{1}{2}(\Delta c + c_u), & \text{if } \Delta c \leq -c_u \\
  p_i^*, & \text{if } -c_u < \Delta c < c_s \\
  p_i^* + \frac{1}{2}(\Delta c - c_s), & \text{if } \Delta c \geq c_s 
\end{cases}, i = 1, 2
\]

- We find that the optimal retail prices do not change when the absolute cost disruption \(|\Delta c|\) is sufficiently small.
- The central planner changes the retail prices if the amount of the unit cost is sufficiently large.
- The larger the incremental unit cost \(\Delta c\) the higher the optimal retail prices.
3 Coordination mechanism

3.1 Quantity discount

■ All-unit quantity discount
If order quantity is bigger than the breakpoint, the discount price would be operated for all purchased units.

■ Two-part tariff based on incremental quantity discount
  - Two-part tariff: fixed fee & constant marginal cost
  - Incremental quantity discount: charging different discount for different order quantity.
3 Coordination mechanism

3.2 Coordination via all-unit quantity discount scheme

- Coordination under cost disruption

  General situation

  When retailer $i$ gets a wholesale price $\tilde{\alpha}_i$, his profit function is

  $$\tilde{\pi}_i(\tilde{p}_i, \tilde{p}_j, \tilde{w}_j) = (\tilde{p}_i - c_i - \tilde{w}_i)(a - \tilde{p}_i + d \tilde{p}_j).$$  \hspace{1cm} (6)

  Solving the first-order conditions of (6) with respect to retail prices, we obtain Nash equilibrium prices

  $$\tilde{p}^*_i = \frac{2(a + c_i + \tilde{w}_i) + d(a + c_j + \tilde{w}_j)}{4 - d^2}. \hspace{1cm} (7)$$

  The supply chain is coordinated only when the Nash equilibrium retail prices for the decentralized supply chain equal the optimal retail prices for the centralized supply chain, respectively: $\tilde{p}^*_N = \tilde{p}^*$, $i = 1, 2$, so we get

  $$\tilde{w}^*_i = \tilde{p}^*_i - c_i - \tilde{q}^*_i = \frac{a(1 + d) - (2 - d^2)\tilde{q}^*_i - d\tilde{q}^*_j}{1 - d^2} - c_i. \hspace{1cm} (8)$$

  Rewriting the profit function of retailer $i$ we have

  $$\tilde{\pi}_i(\tilde{q}_1, \tilde{w}_1, \tilde{q}_2, \tilde{w}_2) = \left(\frac{a(1 + d) - \tilde{q}_1 - d\tilde{q}_j}{1 - d^2} - c_i - \tilde{w}_1\right)\tilde{q}_1. \hspace{1cm} (9)$$
3 Coordination mechanism

3.2 Coordination via all-unit quantity discount scheme

Coordination under cost disruption

- General situation

\[ \tilde{q}_1, \tilde{q}_2 \] --- 2 breakpoints of all-unit quantity discount scheme, \( 0 < \tilde{q}_1 < \tilde{q}_2 \)

\[ \tilde{\omega}_1, \tilde{\omega}_2 \] --- corresponding unit whole sale price, \( \tilde{\omega}_0 > \tilde{\omega}_1 > \tilde{\omega}_2 \)
3 Coordination mechanism

3.2 Coordination via all-unit quantity discount scheme

- Coordination under cost disruption
  - Symmetric case: $c_1 = c_2$

Theorem 2: Assume $q_1 = q_2$. The supply chain is coordinated by an all-unit quantity
discount scheme with the breakpoint $\tilde{a}_1^* = \tilde{a}_1^*$, the corresponding unit wholesale prices $\tilde{a}_0^*$
(large enough) and $\tilde{a}_1^* = \tilde{a}_1^* = \left[\frac{a - (2 - d)q_1^*}{1 - d}\right] - c_1$, where $s$ is given by (5) with $c_1 = c_2$.

- the supply chain with competing retailers can be coordinated by an all-unit quantity
discount scheme with a unique breakpoint if two retailers are identical.
3 Coordination mechanism

3.2 Coordination via all-unit quantity discount scheme

- Coordination under cost disruption
  - Asymmetric case: \( c_1 > c_2 \)

\[
\begin{align*}
\bar{p}_1(q, \bar{w}_1, \bar{q}_1^*, \bar{w}_2^*) & \leq \bar{p}_1(q, \bar{w}_1, \bar{q}_2^*, \bar{w}_2^*) \\
& \text{for } q \in (\bar{q}_1^*, \bar{q}_2^*), \\
\bar{p}_1(q, \bar{w}_2^*, \bar{q}_2^*, \bar{w}_2^*) & \leq \bar{p}_1(q, \bar{w}_1^*, \bar{q}_2^*, \bar{w}_2^*) \\
& \text{for } q \geq \bar{q}_2^*, \\
\bar{p}_2(\bar{q}_1^*, \bar{w}_1^*, q, \bar{w}_1^*) & \leq \bar{p}_2(\bar{q}_1^*, \bar{w}_1^*, \bar{q}_2^*, \bar{w}_2^*) \\
& \text{for } q \in [\bar{q}_1^*, \bar{q}_2^*), \\
\bar{p}_2(\bar{q}_1^*, \bar{w}_1^*, q, \bar{w}_2^*) & \leq \bar{p}_2(\bar{q}_1^*, \bar{w}_1^*, \bar{q}_2^*, \bar{w}_2^*) \\
& \text{for } q > \bar{q}_2^*. 
\end{align*}
\]

Theorem 3: Assume that \( c_1 > c_2 \). If \( c_1 - c_2 \geq 2\alpha(1 - 2\alpha)\bar{q}^*/(1 + d^2) \) and either
\[
\left\lfloor \frac{2\alpha(1 - 2\alpha)\bar{q}^*/(1 + d^2)}{1 + d^2} \right\rfloor \leq c_1 - c_2 \leq \frac{2\alpha\bar{q}_1^*}{(1 + d^2)(2 - d)} \quad \text{or} \\
\frac{2\alpha\bar{q}_1^*}{(1 + d^2)(2 - d)} < c_1 - c_2 \leq \frac{2\alpha\bar{q}_1^*/(1 + d^2)}{4(1 - d^2)\bar{q}_2^* - 2(2 - d)\bar{q}^*/(1 + d^2)}
\]
then the supply chain is coordinated by the all-unit quantity discount scheme with the breakpoints \( \tilde{q}_i^* = \bar{q}_i^* \), \( i = 1, 2 \) the corresponding unit wholesale prices \( \bar{\omega}_0^* \) (large enough) and \( \tilde{\omega}_i^* = \bar{\omega}_i^* \), \( i = 1, 2 \) where \( \tilde{q}_i^* \) is given by (5) and \( \bar{\omega}_i^* \) is given by (8).
3 Coordination mechanism

3.2 Coordination via all-unit quantity discount scheme

- Coordination under cost & demand disruptions
  - demand disruption: \( \tilde{a}, \tilde{a} = a + \Delta a > \tilde{c}_0 \)

**Theorem 4**: For the case with both cost and demand disruptions, the optimal retail prices of the centralized supply chain are

\[
\tilde{p}_i^* = \begin{cases} 
    p_i^* + \frac{(c_a + \Delta c)\Delta a/2}{2(1-d)} + \frac{\Delta c - \Delta a}{(1-d)} & \text{if } \Delta c - \Delta a/(1-d) \leq -c_u \\
    p_i^* + \frac{\Delta a}{2(1-d)} \leq c_u & \text{if } \Delta c - \Delta a/(1-d) < c_u \leq c_v \\
    q_i^* + \frac{1}{2}(1-d)(\Delta c + c_u) + \frac{1}{2} \Delta a, \Delta c - \Delta a/(1-d) \leq c_v & \text{if } q_i^* - c_u < \Delta c - \Delta a/(1-d) \leq c_v \\
    q_i^* - c_u < \Delta c - \Delta a/(1-d) < c_v \\
    q_i^* - \frac{1}{2}(1-d)(\Delta c - c_v) + \frac{1}{2} \Delta a, \Delta c - \Delta a/(1-d) \geq c_v & \text{if } q_i^* - c_u < \Delta c - \Delta a/(1-d) \leq c_v \\
    \end{cases}
\]

- In general, the larger the market scale, the higher the optimal quantity although the retail prices increase.
- Two retailers evenly share the incremental demand due to the symmetric position.
- Nash equilibrium retail prices of the decentralized supply chain are

\[
\tilde{p}_i = \frac{2(a + \Delta a + c_i + \tilde{w}_i) + d(a + \Delta a + c_j + \tilde{w}_j)}{4 - d^2}.
\]
3 Coordination machanism

3.2 Coordination via all-unit quantity discount scheme

- Coordination under cost & demand disruptions
  - Substituting $a + \Delta a$ for $a$ in
    \[
    \tilde{w}_i^* = \tilde{p}_i^* - c_i - \tilde{q}_i^* = \frac{a(1+d) - (2-d^2)\tilde{q}_i^* - d\tilde{q}_j^*}{1-d^2} - c_i. \tag{8}
    \]
    \[
    \tilde{\pi}_i(\tilde{q}_i, \tilde{w}_1, \tilde{q}_2, \tilde{w}_2) = \left(\frac{a(1+d) - \tilde{q}_i - d\tilde{q}_j}{1-d^2} - c_i - \tilde{w}_i\right)\tilde{q}_i. \tag{9}
    \]
    And similar with Theorem 2, we got coordination machanism as follow:

- Symmetric case: $c_1 = c_2$

Theorem 5: Assume $c_2 = c_1$. For the case with cost and demand disruptions, the supply chain is coordinated by an all-unit quantity discount scheme with the breakpoint $\tilde{q}_1^* = \tilde{q}_2^*$, the corresponding unit wholesale prices $\tilde{a}_0^*$, and $\tilde{a}_1^* = \tilde{a}_1^* = \frac{\alpha + \Delta \alpha - (2-d\tilde{q}_1^*)/(1-d)}{1-d} - c_1$. Where $\tilde{q}_1^*$ is given by (14) with $c_2 = c_1$. 
3 Coordination mechanism

3.3 Coordination via incremental quantity discount scheme

- Coordination under cost disruption
  - Two-part tariff: \(((\tilde{\omega}_1^*, \tilde{T}_1^*), (\tilde{\omega}_2^*, \tilde{T}_2^*))\)

  \((\tilde{\omega}_i^*, \tilde{T}_i^*)\) --- the optimal tariff for retailer i.
  \(\tilde{\omega}_i^*\) --- marginal price
  \(\tilde{T}_i^*\) --- fixed fee

The profit of retailer i

\[
\tilde{z}_i = (\tilde{p}_i - c_i - \tilde{w}_i)(a - \tilde{p}_i + d\tilde{p}_j) - \tilde{T}_i, \quad (16)
\]

Nash equilibrium prices

\[
\tilde{p}_i^N = \frac{2(a + c_i + \tilde{w}_i) + d(a + c_j + \tilde{w}_j)}{4 - d^2}. \quad (7)
\]

If the supply chain is coordinated,

\[
\tilde{p}_i^N = \tilde{p}_i^*, \quad i = 1, 2
\]
3 Coordination mechanism

3.3 Coordination via incremental quantity discount scheme

- Coordination under cost disruption
  - Theorem 6

Theorem 6: If the supply chain is coordinated, then the incremental and the all-unit quantity discount schemes have the same marginal wholesale prices $\tilde{\omega}_i^*, \ i=1,2$

The marginal wholesale prices $\tilde{\omega}_i^*$ equal the optimal transfer prices in an centralized supply chain. The central decision-maker takes the marginal wholesale prices $\tilde{\omega}_i^*$. 
3 Coordination mechanism

3.3 Coordination via incremental quantity discount scheme

- Coordination under cost disruption
  - Two-part tariff: \( \{ (\tilde{\omega}_1^*, \tilde{T}_1^*), (\tilde{\omega}_2^*, \tilde{T}_2^*) \} \)

  \( (\tilde{\omega}_i^*, \tilde{T}_i^*) \) --- the optimal tariff for retailer \( i \).

  \( \tilde{\omega}_i^* \) --- marginal price
  \( \tilde{T}_i^* \) --- fixed fee

The profit of retailer \( i \)

\[
\tilde{\pi}_i = (\tilde{p}_i - c_i - \tilde{w}_i)(a + \tilde{p}_i + d\tilde{p}_j) - \tilde{T}_i,
\]

(16)

Nash equilibrium prices

\[
\tilde{p}_i^N = \frac{2(a + c_i + \tilde{w}_i) + d(a + c_j + \tilde{w}_j)}{4 - d^2}.
\]

(7)

\[
\tilde{\pi}_i(\tilde{\omega}_1, \tilde{T}_1, \tilde{\omega}_2, \tilde{T}_2) = \left( (2 - d^2) \tilde{\omega}_1 \right)^2 - \tilde{T}_i
\]

(17)

if both retailers accept the optimal marginal wholesale prices:

\[
\tilde{\pi}_i(\tilde{\omega}_1, \tilde{T}_1, \tilde{\omega}_2, \tilde{T}_2) = \tilde{q}_i^* - \tilde{T}_i^*.
\]
3 Coordination mechanism

3.3 Coordination via incremental quantity discount scheme

- Coordination under cost disruption

  - Constrains of two-part tariff:

    Incentive compatibility constraint: neither retailer has a motivation to unilaterally deviate from the optimal tariffs of the supply chain.

    \[
    \bar{\pi}_1(\bar{w}_2^*, \bar{T}_2^*, \bar{\bar{w}}_2^*, \bar{T}_2^*) \leq \bar{\pi}_1(\tilde{w}_1^*, \tilde{T}_1^*, \tilde{\bar{w}}_2^*, \tilde{T}_2^*), \tag{18}
    \]

    \[
    \bar{\pi}_2(\tilde{w}_1^*, \tilde{T}_1^*, \tilde{\bar{w}}_1^*, \tilde{T}_1^*) \leq \bar{\pi}_2(\tilde{w}_1^*, \tilde{T}_1^*, \tilde{\bar{w}}_2^*, \tilde{T}_2^*). \tag{19}
    \]

  Individual rationality (IR) constraint: retailers would like to order and sell the manufacturer’s products.

    \[
    \tilde{g}_1^{x_2} - \tilde{T}_1^* = \bar{\pi}_1(\tilde{w}_1^*, \tilde{T}_1^*, \tilde{\bar{w}}_2^*, \tilde{T}_2^*) \geq \bar{\pi}_1, \tag{20}
    \]

    \[
    \tilde{g}_2^{x_2} - \tilde{T}_2^* = \bar{\pi}_2(\tilde{w}_1^*, \tilde{T}_1^*, \tilde{\bar{w}}_2^*, \tilde{T}_2^*) \geq \bar{\pi}_2, \tag{21}
    \]

    \[
    \bar{\pi}_i \geq 0 \quad \text{--- reservation utility of retailer } i
    \]
3 Coordination machanism

3.3 Coordination via incremental quantity discount scheme

- Coordination under cost disruption

According to (17)(18)(19)

\[ \pi_i(\tilde{q}_1, \tilde{t}_1, \tilde{q}_2, \tilde{t}_2) = \left( \frac{(2+d)\alpha_i - (2-d^2)(\tilde{c}_i + \tilde{q}_1) + d(\tilde{c}_j + \tilde{q}_j)}{(4-d^2)^2} \right)^2 - \tilde{t}_i \]  \hspace{1cm} (17)

\[ \tilde{q}_2^2 - \tilde{t}_i \]

\[ \tilde{\pi}_1(\tilde{w}_1, \tilde{t}_1, \tilde{w}_2, \tilde{t}_2) \leq \tilde{\pi}_1(\tilde{w}_1^*, \tilde{t}_1^*, \tilde{w}_2^*, \tilde{t}_2^*) \] \hspace{1cm} (18)

\[ \tilde{\pi}_2(\tilde{w}_1^*, \tilde{t}_1^*, \tilde{w}_2^*, \tilde{t}_2^*) \leq \tilde{\pi}_2(\tilde{w}_1^*, \tilde{t}_1^*, \tilde{w}_2^*, \tilde{t}_2^*) \] \hspace{1cm} (19)

we got

\[ \tilde{\pi}_1(\tilde{w}_1^*, \tilde{t}_1^*, \tilde{w}_2^*, \tilde{t}_2^*) \leq \tilde{\pi}_2(\tilde{w}_1^*, \tilde{t}_1^*, \tilde{w}_2^*, \tilde{t}_2^*) \]

It follows from \( \tilde{\omega}_1^* \geq \tilde{\omega}_2^* \) that the two sides of inequality are nonnegative.
3 Coordination machanism

3.3 Coordination via incremental quantity discount scheme

Coordination under cost disruption

- Theorem 7

Theorem 7: The menu of two-part tariffs \( \{ (\alpha_i^*, T_i^*), (\alpha_j^*, T_j^*) \} \) can coordinate the supply chain if the mechanism satisfies (20)–(22).

- For the symmetric case, marginal wholesale prices are equal if and only if the two retailers are identical. When the retailers are identical, both sides of (22) equal zero. 
  \[ \tilde{T}_1^* = \tilde{T}_2^* \] , which means same tariffs .

- For the asymmetric case, the higher marginal wholesale price goes with the lower fixed fee in the coordinating menu of two-part tariffs.
4 Conclusions

- For the all-unit quantity discount scheme, the supply chain can be fully coordinated when two retailers are identical.

- For the coordinating menu of two-part tariffs, under the original mechanism, the lower-cost retailer accepts the original wholesale price, but the higher-cost retailer may have a motivation to order more products to accept the lower wholesale price.