Introduction to Undercut-proof Equilibrium (with example)

2017.03.15
Yunhyoung Kim
Table of Contents

• Introduction
• Undercut-proof Equilibrium
• Application
• Summary and Conclusion
Introduction

Describing “competition” in a market

Winner Takes It All: Bertrand/Cournot Competition
Market Division: Hotelling’s Model / Logit Model
Introduction

Describing “competition” in a market

Winner Takes It All: Bertrand/Cournot Competition
Market Division: Hotelling’s Model / Logit Model

Undercut-Proof Equilibria
by Peter B. Morgan and Oz Shy
(SUNY Buffalo / U. of Haifa)

← Cited 32 times
Introduction

Describing “competition” in a market

Winner Takes It All: Bertrand/Cournot Competition
Market Division: Hotelling’s Model / Logit Model

Undercut-Proof Equilibria
by Peter B. Morgan and Oz Shy
(SUNY Buffalo / U. of Haifa)

A Quick-and-easy Method for Estimating Switching Costs
by Oz Shy
Introduction

Simple Explanation on Undercut-proof Equilibria

Undercut-proof Equilibria: an extension of price competition (similar to Bertrand competition)

difference $\rightarrow$ distaste cost
Introduction

Simple Explanation on Undercut-proof Equilibria

Undercut-proof Equilibria: an extension of price competition (similar to Bertrand competition)

difference $\rightarrow$ distaste cost

Example: Music Streaming Service Provider

<table>
<thead>
<tr>
<th>Service</th>
<th>Price</th>
<th>Preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Genie</td>
<td>5500</td>
<td>45% prefers 'genie'</td>
</tr>
<tr>
<td>Melon</td>
<td>4600</td>
<td>55% prefers 'melon'</td>
</tr>
</tbody>
</table>
Introduction

Simple Explanation on Undercut-proof Equilibria

Undercut-proof Equilibria: an extension of price competition (similar to Bertrand competition)

difference $\Rightarrow$ distaste cost

Example: Music Streaming Service Provider

```
<table>
<thead>
<tr>
<th></th>
<th>genie</th>
<th>Melon</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>5500</td>
<td>4600</td>
</tr>
<tr>
<td>P2</td>
<td>45%</td>
<td>55%</td>
</tr>
</tbody>
</table>

Subscribe 'genie' at 5500
$\Rightarrow$ if 4600 < 5500 - S, subscribe 'melon'

S: distaste cost

Subscribe 'melon' at 4600
$\Rightarrow$ if 5500 < 4600 - S, subscribe 'melon'
```
Introduction

Simple Explanation on Undercut-proof Equilibria

Example: Music Streaming Service Provider (cont.)

S: distaste cost = 1000

Even though ‘Melon’ provides the same service for a cheaper price, I’ll choose ‘Genie’.
Even though ‘Melon’ provides the same service for a cheaper price, I’ll choose ‘Genie’.

I do not like ‘Melon’, but I have to choose ‘Melon’ since it is far cheaper.

S : distaste cost = 1000
Introduction

Simple Explanation on Undercut-proof Equilibria

Example: Music Streaming Service Provider (cont.)

Even though 'Melon' provides the same service for a cheaper price, I'll choose 'Genie'.

I do not like 'Melon', but I have to choose 'Melon' since it is far cheaper.

→ 'Melon' monopolizes the market

What happen if 'Genie' undercuts?
Introduction

Simple Explanation on Undercut-proof Equilibria

Example: Music Streaming Service Provider (cont.)

S: distaste cost = 1000

I do not like 'Genie', but I have to choose 'Genie' since it is far cheaper.

→ 'Genie' monopolizes the market

What happens if 'Melon' undercuts?
Introduction

Simple Explanation on Undercut-proof Equilibria

Example: Music Streaming Service Provider (cont.)

\[ \text{S: distaste cost} = 1000 \]

I do not like ‘Genie’, but I have to choose ‘Genie’ since it is far cheaper.

→ ‘Genie’ monopolizes the market

What happen if ‘Melon’ undercuts?

They will keep undercutting until they reach equilibria.

→ Undercut-proof equilibria

What would be the end of this repetition?
Undercut-proof Equilibria

Setting

two stores called $A$ and $B$

<table>
<thead>
<tr>
<th>Store A</th>
<th>Store B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_A$</td>
<td>$n_B$</td>
</tr>
<tr>
<td>number of consumers</td>
<td>number of consumers</td>
</tr>
</tbody>
</table>

two groups of consumers

<table>
<thead>
<tr>
<th>type $\alpha$</th>
<th>type $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_A &gt; 0$</td>
<td>$N_B &gt; 0$</td>
</tr>
</tbody>
</table>
Undercut-proof Equilibria

Setting

two stores called $A$ and $B$

Store $A$  
$\uparrow n_A$
number of consumers

Store $B$  
$\uparrow n_B$

two groups of consumers

type $\alpha$

$N_A > 0$

type $\beta$

$N_B > 0$

\[ U_\alpha \overset{\text{def}}{=} \begin{cases} -p_A & \text{staying with brand A} \\ -p_B - S & \text{switching to brand B} \end{cases} \]

\[ U_\beta \overset{\text{def}}{=} \begin{cases} -p_A - S & \text{switching to brand A} \\ -p_B & \text{staying with brand B} \end{cases} \]

$\rightarrow$ Discrete Version of Hotelling's Model
Undercut-proof Equilibria

Setting

two stores called \( A \) and \( B \)

- \( n_A \), number of consumers
- \( n_B \), number of consumers

\[
\begin{align*}
n_A &= \begin{cases}
0 & \text{if } p_A > p_B + S \\
\frac{N_A}{N_A + N_B} & \text{if } p_B - S \leq p_A \leq p_B + S, \\
\frac{N_A}{N_A + N_B} & \text{if } p_A < p_B - S
\end{cases} \\
n_B &= \begin{cases}
0 & \text{if } p_B > p_A + S \\
\frac{N_B}{N_A + N_B} & \text{if } p_A - S \leq p_B \leq p_A + S, \\
\frac{N_B}{N_A + N_B} & \text{if } p_B < p_A - S.
\end{cases}
\end{align*}
\]

two groups of consumers

- \( N_A > 0 \)
- \( N_B > 0 \)

- \( \alpha \) type
- \( \beta \) type

\[
\begin{align*}
U_\alpha \overset{\text{def}}{=} \begin{cases}
-p_A & \text{staying with brand A} \\
-p_B - S & \text{switching to brand B}
\end{cases} \\
U_\beta \overset{\text{def}}{=} \begin{cases}
-p_A - S & \text{switching to brand A} \\
-p_B & \text{staying with brand B}
\end{cases}
\end{align*}
\]

\( \rightarrow \) Discrete Version of Hotelling's Model
Undercut-proof Equilibria

Setting

two stores called $A$ and $B$

Store A  

$\uparrow$

number of consumers

$n_A = \begin{cases} 
0 & \text{if } p_A > p_B + S \rightarrow B \text{ monopolizes} \\
N_A & \text{if } p_B - S \leq p_A \leq p_B + S, \\
N_A + N_B & \text{if } p_A < p_B - S \rightarrow A \text{ monopolizes} 
\end{cases}$

Store B  

$\uparrow$

$n_B = \begin{cases} 
0 & \text{if } p_B > p_A + S \rightarrow A \text{ monopolizes} \\
N_B & \text{if } p_A - S \leq p_B \leq p_A + S, \\
N_A + N_B & \text{if } p_B < p_A - S. \rightarrow B \text{ monopolizes} 
\end{cases}$

two groups of consumers

$N_A > 0$  

$N_B > 0$

discrete Version of Hotelling’s Model

$U_\alpha \overset{\text{def}}{=} \begin{cases} 
-p_A & \text{staying with brand A} \\
-p_B - S & \text{switching to brand B} 
\end{cases}$

$U_\beta \overset{\text{def}}{=} \begin{cases} 
-p_A - S & \text{switching to brand A} \\
-p_B & \text{staying with brand B} 
\end{cases}$
Definition

Definition 2. A pair of prices $\langle p_A^U, p_B^U \rangle$ is said to satisfy the **Undercut-proof Property (UPP)** if

(a) For given $p_B^U$ and $n_B^U$, firm $A$ chooses the highest price $p_A^U$ subject to

$$\pi_B^U = p_B^U n_B^U \geq (p_A - S)(N_A + N_B).$$

(b) For given $p_A^U$ and $n_A^U$, firm $B$ chooses the highest price $p_B^U$ subject to

$$\pi_A^U = p_A^U n_A^U \geq (p_B - S)(N_A + N_B).$$
Definition 2. A pair of prices $\langle p_A^U, p_B^U \rangle$ is said to satisfy the **Undercut-proof Property (UPP)** if

(a) For given $p_B^U$ and $n_B^U$, firm $A$ chooses the highest price $p_A^U$ subject to

$$\pi_B^U = p_B^U n_B^U \geq (p_A - S)(N_A + N_B).$$  \(\Rightarrow\) **Highest price satisfying “B do not undercuts A”**

(b) For given $p_A^U$ and $n_A^U$, firm $B$ chooses the highest price $p_B^U$ subject to

$$\pi_A^U = p_A^U n_A^U \geq (p_B - S)(N_A + N_B).$$  \(\Rightarrow\) **Highest price satisfying “A do not undercuts B”**
Undercut-proof Equilibria

Definition

Definition 2. A pair of prices \( \langle p^U_A, p^U_B \rangle \) is said to satisfy the **Undercut-proof Property (UPP)** if

(a) For given \( p^U_B \) and \( n^U_B \), firm A chooses the highest price \( p^U_A \) subject to

\[
\pi^U_B = p^U_B n^U_B \geq (p_A - S)(N_A + N_B). \quad \Rightarrow \text{Highest price satisfying “B do not undercuts A”}
\]

(b) For given \( p^U_A \) and \( n^U_A \), firm B chooses the highest price \( p^U_B \) subject to

\[
\pi^U_A = p^U_A n^U_A \geq (p_B - S)(N_A + N_B). \quad \Rightarrow \text{Highest price satisfying “A do not undercuts B”}
\]

Condition that “prevents the other firm from undercutting”

: no difference before/after undercutting

\( \Rightarrow \) **monopoly price does not give more profit than undercut-proof price**

\[
\begin{align*}
p^U_A n^U_A &= (p_B - S)(N_A + N_B) \\
p^U_B n^U_B &= (p_A - S)(N_A + N_B)
\end{align*}
\]

\[
\begin{align*}
p^U_A &= \frac{(N_A + N_B)(N_A + 2N_B)S}{(N_A)^2 + N_A N_B + (N_B)^2} \quad \text{and} \quad p^U_B &= \frac{(N_A + N_B)(2N_A + N_B)S}{(N_A)^2 + N_A N_B + (N_B)^2}
\end{align*}
\]
**Application**

**Terrestrial Broadcaster and Pay TVs**

**Purpose**: Study the impact of terrestrial network coverage

1) Change of profits
2) Effect on retransmission fee

2 competitions:
1. Terrestrial Broadcaster vs. Pay TV
2. Pay TV A vs. Pay TV B
1. Terrestrial Broadcaster vs. Pay TV

- **Watch Terrestrial TV Programs**
  - Terrestrial TV Viewer: $v_B \cdot \theta$
  - Pay TV Viewer: $(v_P + v_B) \cdot \theta - p$

- **Subscribe to Pay TV**
  - Viewer: $\theta$
  - Viewers’ degree of liking

Inside network coverage $\alpha$
Application

Terrestrial Broadcaster and Pay TVs

1. Terrestrial Broadcaster vs. Pay TV

Watch Terrestrial TV Programs

Terrestrial TV Viewer $v_B \cdot \theta$

Subscribe to Pay TV

Pay TV Viewer $(v_P + v_B) \cdot \theta - p$

Inside network coverage $\alpha$

Outside of network coverage $1 - \alpha$

Do not Watch

Do not watch 0

Subscribe to Pay TV

Pay TV Viewer $(v_P + v_B) \cdot \theta - p$
Application

Terrestrial Broadcaster and Pay TVs

1. Terrestrial Broadcaster vs. Pay TV

Inside network coverage

Outside of network coverage

Viewers’ Utility

$\frac{p}{v_P}$

Terrestrial TV Viewer

Pay TV Viewer

Do not watch

Pay TV Viewer
1. Terrestrial Broadcaster vs. Pay TV

**Inside network coverage**

- **Terrestrial TV Viewer**
- **Pay TV Viewer**

**Outside of network coverage**

- **Do not watch**
- **Pay TV Viewer**

\[ N_B = \alpha \cdot \min \left[ \frac{p}{v_p}, 1 \right] \]

where

\[ N_\alpha^P = \begin{cases} 
1 - \frac{p}{v_p}, & \text{if } \frac{p}{v_p} \leq 1 \\
0, & \text{otherwise}
\end{cases} \]

\[ N_{1-\alpha}^P = \begin{cases} 
1 - \frac{p}{v_p + v_B}, & \text{if } \frac{p}{v_p + v_B} \leq 1 \\
0, & \text{otherwise}
\end{cases} \]
Application

Terrestrial Broadcaster and Pay TVs

2. Pay TV A vs. Pay TV B

Only One Pay TV
⇒ Optimal price is not realistic

\[ p^*(\alpha, c) = \frac{c}{2} + \frac{v_P(v_B + v_P)}{2(\alpha \cdot v_B + v_P)} \]

\[ N_P(p^*) = 1/2 \]
Application

Terrestrial Broadcaster and Pay TVs

2. Pay TV A vs. Pay TV B

Assumption: A and B are identical
\[ N_A(p) = N_B(p) = 0.5 \cdot N_p(p), \quad p^u_A = p^u_B = p^u \]

\[ p^u_A n_A^U = (p_B - S)(N_A + N_B) \]

\[ p^u_B n_B^U = (p_A - S)(N_A + N_B) \]
Application

Terrestrial Broadcaster and Pay TVs

2. Pay TV A vs. Pay TV B

Assumption: A and B are identical

\[ N_A(p) = N_B(p) = 0.5 \cdot N_P(p), \quad p^A = p^B = p^u \]

\[ p^U_A n_A^U = (p_B - S)(N_A + N_B) \]
\[ p^U_B n_B^U = (p_A - S)(N_A + N_B) \]

\[ (p^u - c) \cdot \frac{1}{2} N_P(p^u) = (p^u - S - c) \cdot N_P(p^u - S) \]
Application

Terrestrial Broadcaster and Pay TVs

2. Pay TV A vs. Pay TV B

Assumption: A and B are identical
\[ N_A(p) = N_B(p) = 0.5 \cdot N_P(p), \quad p_A^u = p_B^u = p^u \]

\[ p_A^U n_A^U = (p_B - S)(N_A + N_B) \]
\[ p_B^U n_B^U = (p_A - S)(N_A + N_B) \]

\[ (p^u - c) \cdot \frac{1}{2} N_P(p^u) = (p^u - S - c) \cdot N_P(p^u - S) \]

\[ p^u(\alpha, c) = \frac{c}{2} + \frac{v_P(v_B + v_P)}{2(\alpha v_B + v_P)} + 2S - \sqrt{\left(\frac{v_P(v_B + v_P)}{2(\alpha v_B + v_P)} - \frac{c}{2}\right)^2 + 2S^2} \]

\[ p^u(\alpha, c) = p^* - \tau \]

where \[ \tau = \sqrt{(p^* - c)^2 + 2S^2} - 2S \]

\[ p^*(\alpha, c) = \frac{c}{2} + \frac{v_P(v_B + v_P)}{2(\alpha \cdot v_B + v_P)} \]
Application

Terrestrial Broadcaster and Pay TVs

2. Pay TV A vs. Pay TV B

Difference between $p^u$ and $p^*$

$N_p(p^u) + N_B(p^u)$

$N_p(p^*) + N_B(p^*)$

$N_p(p^u)$: sum of pay TV viewers
Summary and Conclusion

• Undercut-proof equilibria
  – Describes the competitive market situation
  – Based on price competition / discrete version of Hotelling’s model
  – Equilibrium is at a point that prevents the other from undercutting