Assortment and Price Competition with Product Introduction Cost

Jungju Park and Jeonghoon Mo
Information and Industrial Engineering Department, Yonsei University

Working paper

Speaker JUNGJU PARK
Date 31^{th} May 2017

Key words

- Assortment planning
- Pricing
- Competition
- W/ Product introduction cost

- Multinomial logit model (MNL)

- Results on the existence of Equilibrium

Joint assortment and pricing competition

- Firms want to maximize her profit.
  - To configure a product line and to set its elements' prices
  - Competitive business environment

- Besbes and Sauré (2015)
  - **Equilibrium existence:** *There always exists an equilibrium in the assortment-only competition.*
  - **Best equilibrium:** *When there are multiple equilibria, one (the smallest) Pareto-dominates all the others.*

- However, in real world:

Samsung smartphones

VS.

Apple smartphone

Model: Endogenous product line size decision

- Notations
  - $\mathcal{J} = \{1, 2\}$ set of firms
  - $\mathcal{R}_j = \{1, 2, \dots, R_j\}, j \in \mathcal{J}$, set of potential products of firm j
  - v_{jr}, product value of firm j's product r
  - c_{jr}, marginal cost of firm j's product r
  - f_{jr}, marginal cost of firm j's product r
  - b, consumer price sensitivity

- Payoffs under the MNL is:

$$\pi_j(\mathcal{R}_j, \boldsymbol{p}_j | \mathcal{R}_{j'}, \boldsymbol{p}_{j'}) = \sum_{r \in \mathcal{R}_j} \{(p_{jr} - c_{jr}) s_{jr}(\mathcal{R}_1, \boldsymbol{p}_1, \mathcal{R}_2, \boldsymbol{p}_2) - f_{jr}\} x_{jr}$$

where

$$s_{jr}(\mathcal{R}_1, \boldsymbol{p}_1, \mathcal{R}_2, \boldsymbol{p}_2) = \frac{u_{jr}}{1 + \sum_{j' \in \mathcal{J}} \sum_{r' \in \mathcal{R}_{j'}} u_{j'r'} x_{j'r'}}$$

and $u_{ni} = \exp(v_{ni} - b p_{ni}), j \in \{1, 2\}, r \in \mathcal{R}_j$.

연세대학교 YONSEI UNIVERSITY

NEtwork media Management & Optimization

Description of the game

- Players: firm $j \in \{1, 2\}$.
- Strategy: $\boldsymbol{x}_j$, product line; $\boldsymbol{p}_j$, price.
- Payoffs: $\pi_j\left(\mathcal{R}_j, \boldsymbol{p}_j \,|\mathcal{R}_{j'}, \boldsymbol{p}_{j'}\right), j \neq j'$.

- Order of the game: Complex sequential game
  - Stage 1: Simultaneously, both firms determine their product lines.
  - Stage 2: Known each firm's product line, both firms simultaneously determine their product prices.

Analysis (1/3): Optimal price characterization

- The first order condition of $\pi_j(\mathcal{R}_j, \boldsymbol{p}_j | \mathcal{R}_{j'}, \boldsymbol{p}_{j'})$ is a sufficient condition.
  - Sketch of proof. We show that the objective function of $\pi_j(\mathcal{R}_j, \boldsymbol{p}_j | \mathcal{R}_{j'}, \boldsymbol{p}_{j'})$ is uni-modal with respect to p_{jr}.

- According to FOC, the optimal price $\boldsymbol{p}_j$ verifies that:

$$p^*_{jr} = c_{jr} + \frac{1}{b}\left(1 + \mathrm{W}\Big(\frac{1}{K_j}\sum_{r' \in \mathcal{R}_j} a_{jr'} x_{jr'}\Big)\right), \; j \in \mathcal{J},\; r \in \mathcal{R}_j$$

  where W(.) is the Lambert W function, and $a_{jr} = \exp(v_{jr} - bc_{jr} - 1)$.
  - Note that the equation is in implicit form.

- There exists a unique Nash price equilibrium (Konovalov and sndor, 2010).

Analysis (2/3): Optimal price characterization

- We plug the optimal price equations

$$p_{jr}^* = c_{jr} + \frac{1}{b}\left(1 + \mathrm{W}\Big(\frac{1}{K_j}\sum_{r'\in\mathcal{R}_j} a_{jr'}x_{jr'}\Big)\right), \; j \in \mathcal{J},\, r \in \mathcal{R}_j$$

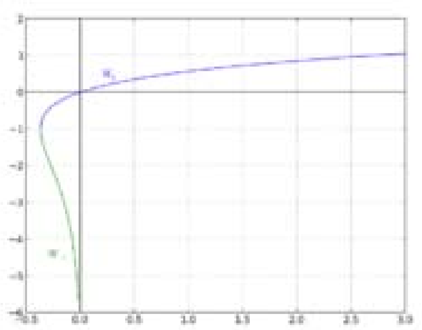
  into the original payoff function $\pi_j(\mathcal{R}_j, \boldsymbol{p}_j | \mathcal{R}_{j'}, \boldsymbol{p}_{j'})$.

- Complicated, but finally we have the price-optimized product line selection problem that:

$$\Phi_j(\mathbf{x}_j, | \mathbf{x}_{-j}) = \frac{Q}{b}\mathrm{W}\Big(\frac{1}{K_j(\mathbf{x}_{-j})}\sum_{r\in\mathcal{R}_j} a_{jr}x_{jr}\Big) - \sum_{r\in\mathcal{R}_j} f_{jr}x_{jr}$$

  where $K_j(\boldsymbol{x}_{-j}) = 1 + \sum_{j'\in\mathcal{J}\setminus\{j\}}\sum_{r'\in\mathcal{R}_{j'}} u_{j'r'}x_{j'r'}$.

  – Note that the first-term represents marginal profit and the second-term does the net fixed costs.



Analysis (3/3): Optimal product line characterization

- WLOG, we expand the problem.
  - Let firm j has H_j different levels of fixed cost ($1 \leq H_j \leq R_j$).
  - In addition, $\mathcal{R}_{jh} = \{1, 2, \dots, R_{jh}\}$ such that $\cup_{h \in H_j} \mathcal{R}_{jh} = \mathcal{R}_j$.
  - We have the following equivalent problem:

$$\Phi_j(\mathbf{x}_j, | \mathbf{x}_{-j}) = \frac{Q}{b} \mathrm{W}\Big(\frac{1}{K_j(\mathbf{x}_{-j})} \sum_{h \in \mathcal{H}_j} \big(\sum_{r \in \mathcal{R}_{jh}} a_{jhr} x_{jhr} \big)\Big) - \sum_{h \in \mathcal{H}_j} (f_{jh} \sum_{r \in \mathcal{R}_{jhr}} x_{jhr});$$

- Optimal solution verifies that for each $\mathcal{R}_{jh}$, the optimal product line has only the highest-valued products.
  - Example. $\mathcal{R}_{j1} = \{\mathbf{5}, \mathbf{4}, 3, 2, 1\}$, $\mathcal{R}_{j2} = \{\mathbf{5}, \mathbf{4}, \mathbf{3}, 2, 1\}$