Pricing Multiple Products with the Multinomial Logit and Nested Logit Models: Concavity and Implications

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Manufacturing & Service Operations Management
Vol. 13, No. 4, Fall 2011, pp. 549-563

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Date 12^{th} April 2017

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NEtwork media Management & Optimization

About pricing

- *Needless to say, <u>pricing is important</u>.*

- Complexity of pricing to consider ("Pricing" of Wikipedia):
  - the financial goals of the company (i.e. profitability),
  - the fit with marketplace realities (will customers buy at that price?),
  - the extent to which the price supports a product's market positioning and be consistent with the other variables in the marketing mix.

- The Hotelling model discusses:
  - Influence on differentiation
  - Impact of product value
  - Impact of marginal cost
  - Impact of demand
  - Impact of competing firm

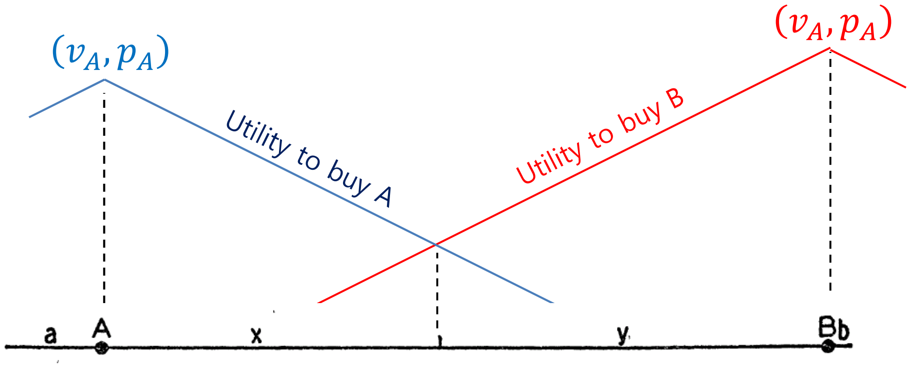


FIG. 1.

Pricing multiple products and research question

- Firms used to have multiple products with different values.

- Research questions:
  - Will there be an equilibrium for the competition of a pricing multiple product game (under Nested logit model)?
  - What will be the difference between the monopoly and the oligopoly market?

- Complexity on pricing multiple products
  - Coexistence of cannibalization and competing products

Payoff under the Nested logit model

- Optimization model:
  - Two firms $k \in \{1, 2\}$ as the players
  - To determine R_k products' prices of $\boldsymbol{p}_k = \{p_{k1}, p_{k2}, \ldots, p_{kR_k}\}$
  - The payoff is:

$$\pi_k(\boldsymbol{p}_1, \boldsymbol{p}_2) = \sum_r (p_{kr} - c_{kr}) q_{kr}(\boldsymbol{p}_1, \boldsymbol{p}_2)$$

where $q_{jk}(\boldsymbol{p}_1, \boldsymbol{p}_2)$ is the sales amount of product j in nest k.

- Nested logit model (Generalized Multinomial logit model)
  - Hierarchical choice (nest first ➔ product second)
  - τ_k, substitution parameter for the products within nest k

$$q_{jk}(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{\left(\sum_{j=1}^{R_k} \exp\left(v_{jk'} - b_k p_{jk'}\right)\right)^{\tau_k}}{1 + \sum_{j'} \left(\sum_{j=1}^{R_k} \exp\left(v_{jk'} - b_k p_{jk'}\right)\right)^{\tau_{k'}}} \frac{\exp\left(v_{jk} - b_j p_{jk}\right)}{1 + \sum_{j=1}^{R_{k'}} \exp\left(v_{jk'} - b_k p_{jk'}\right)}$$

Nest selection   Product selection

Monopoly solution

- Lemma. There exists one-to-one mapping from a price vector $\boldsymbol{p}$ to a quantity vector $\boldsymbol{q}$.

- Theorem 1. $\max_{\boldsymbol{q}_j} \pi_j(\boldsymbol{q}_j)$ is a convex problem where:

$$\pi_j(\boldsymbol{q}_j) = \sum_{k=1}^{K}\sum_{j=1}^{M_k} \frac{a_{jk}}{b_k} \cdot q_{jk} + \sum_{k=1}^{K}\sum_{j=1}^{M_k} \frac{1}{b_k} \cdot q_{jk} \left[\log\left(1 - \sum_l Q_l\right) - \log q_{jk} \right] + \sum_{k=1}^{K}\sum_{j=1}^{M_k} \frac{1}{b_k} \frac{1-\tau_k}{\tau_k} \cdot q_{jk} \left[\log\left(1 - \sum_l Q_l\right) - \log Q_k \right].$$

- Theorem 2. The optimal expected profit π^* is:

$$\pi^* = \sum_k \frac{e^{-b_r \tau_k \pi^*}}{b_k \tau_k} \left(e^{-1} \sum_j e^{a_{jk} - b_k c_{jk}} \right)^{\tau_k}$$

where $p_{jk}^* = \pi^* + \frac{1}{b_k \tau_k} + c_{jk}$.

Oligopoly solution: Quantity vs. price competitions

- **Quantity competition (Theorem 3)**
  - Equilibrium price of product j in nest k is:

$$\hat{p}_{jk} = \frac{1 + W(\bar{A}_k)}{b_k \tau_k} + c_{jk}, \quad and$$

$$\hat{q}_{jk} = \hat{Q}_k \cdot \frac{e^{a_{jk} - b_k c_{jk}}}{\sum_{l=1}^{M_k} e^{a_{lk} - b_k c_{lk}}}, \quad where\ \hat{Q}_k = \frac{W(\bar{A}_k)}{1 + \sum_{l=1}^{K} W(\bar{A}_l)}.$$

where $W(x)$ is the inverse function of $y = xe^x$.

- **Price competition (Theorem 4)**
  - Equilibrium price of product j in nest k is:

$$\tilde{p}_{jk} = \frac{1}{b_k \tau_k} \cdot \frac{1}{1 - V(\bar{A}_k \tilde{Q}_0)} + c_{jk}, \quad and$$

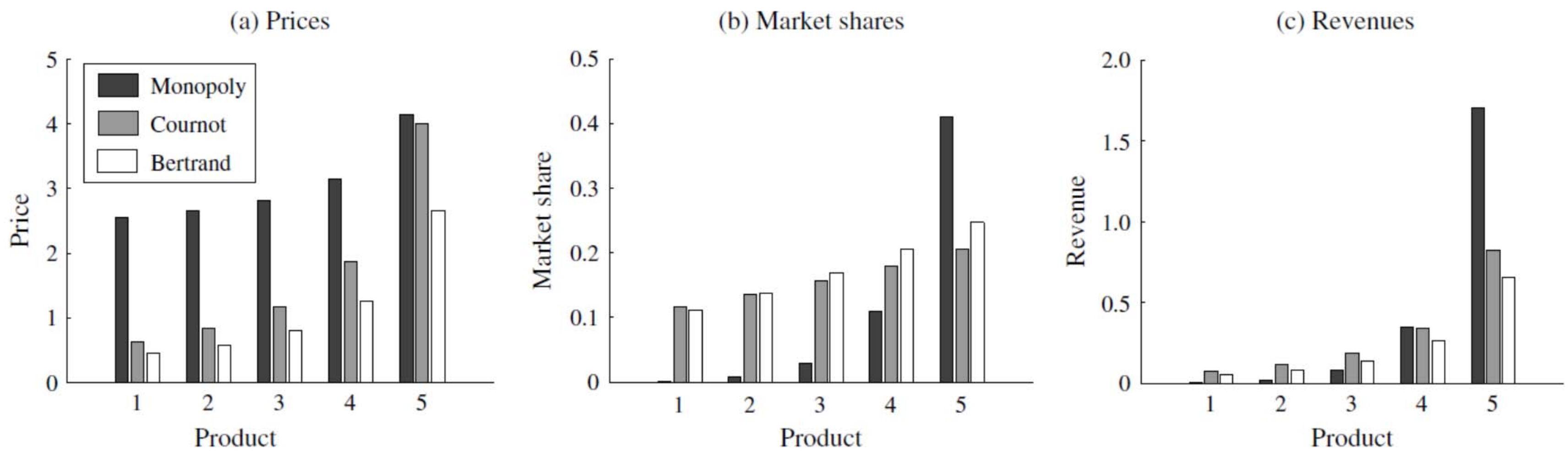
$$\tilde{q}_{jk} = \tilde{Q}_k \cdot \frac{e^{a_{jk} - b_k c_{jk}}}{\sum_{l=1}^{M_k} e^{a_{lk} - b_k c_{lk}}}, \quad where\ \tilde{Q}_k = V(\bar{A}_k \tilde{Q}_0).$$

where $V(x)$ is the inverse function of $y = xe^{\left(\frac{x}{1-x}\right)}$.

Monopoly vs. oligopoly markets

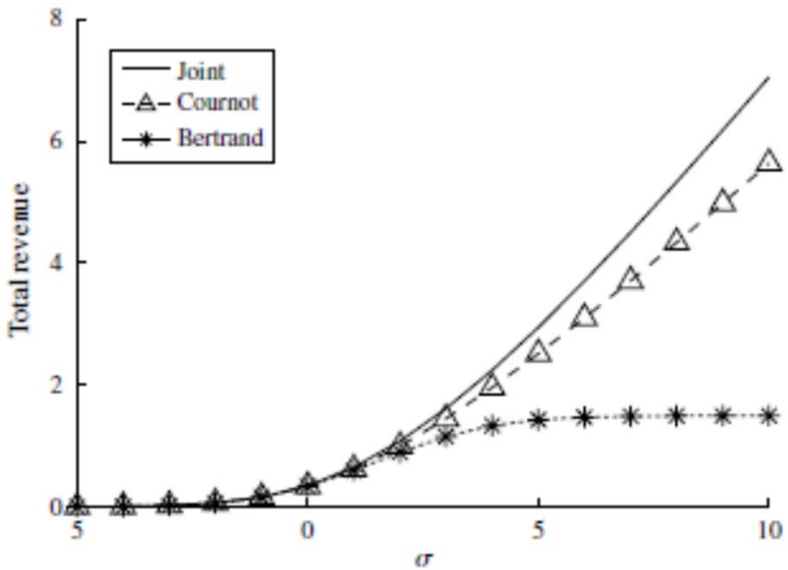
- **Price comparison:** $p^*_{jk} \geq \hat{p}_{jk} \geq \tilde{p}_{jk}$.
  - Consistent with the insights obtained from incumbent studies.

- **E.g., Oligopoly case with single product firms $\boldsymbol{k} \in \{\mathbf{1}, \ldots, \mathbf{5}\}$**
  - $v_k < v_{k+1}$, $b_k > b_{k+1}$, $c_k = 0$
  - Figure 1: $p^*_{jk} \geq \hat{p}_{jk} \geq \tilde{p}_{jk}$.
  - Figure 2 and 3: High value and low sensitivity dominates the others. (while competition increases product variety in the market.)



Additional notes

- **Joint vs. decentralized pricing**
  - Decentralization means each firm unit compete with each other.
  - $\pi_{joint} \geq \pi_{Cournot} \geq \pi_{Bertrand}$ (below Figure)



- **Application to joint inventory-pricing control**
  - Make-to-assemble, make-to-stock, and non-replenishable cases
  - (omitted in this slides)

Conclusions

- Studied a pricing competition under the Nested logit model.
- The price for any individual product is always the highest under the monopoly and the lowest under the price-competition.
- Products with stronger brand power will have a smaller market share under competition than under a monopoly.
- Profit loss due to decentralized decision is greater under the price-competition oligopoly than under the quantity-competition oligopoly.