Product Assortment and Price Competition under Multinomial Logit Model

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Key words

- Assortment planning (=product line selection)
- Pricing

- Multinomial logit model (MNL)
- Capability to produce the same product
- Cardinality-constrained

- Competition
- Results on the existence of Equilibrium

Joint product line and pricing competition

- Firms want to maximize her profit.
- To configure a product line and to set its elements' prices
- Competitive business environment

- Research goal is to study equilibrium behavior by:
  - ***To show the existence of equilibrium***, and
  - ***To analyze the impacts of the cardinality*** *(vs. inventory)* ***limit*** *and the existence of the common products (not in this presentation)*

Potential product set of firm n

Chosen product set w/ $\boldsymbol{p_n} = \{p_{ni}\}$

Game model (No common product)

- Two firms $n \in \{1, 2\}$ as the players

- Strategy of each firm is $\{A_n, \boldsymbol{p}_n\}$
  - $A_n = \{A \subseteq S_n : A \leq C_n\}$, chosen (potential) product set
  - $\boldsymbol{p}_n$, price vector

- To represent their payoffs, the MNL is adopted as:

$$\pi_n = \sum_{i \in A_n} (p_{ni} - c_{ni}) q_{ni}(A_1, \boldsymbol{p}_1, A_2, \boldsymbol{p}_2)$$

where

$$q_{ni}(A_1, \boldsymbol{p}_1, A_2, \boldsymbol{p}_2) = \frac{v_{ni}}{1 + \sum_{i' \in A_1} v_{1,i'} + \sum_{i'' \in A_2} v_{2,i''}}$$

and $v_{ni} = \exp(u_{ni} - \alpha_{ni} p_{ni})\,, n \in \{1, 2\}, i \in A_n$

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Assortment-only competition

- **A partial problem with given prices** $(p_{ni} > c_{ni})$

- Sketch of analysis
  - Provided a tractable equivalent problem as:

$$\max \lambda$$

$$s.t. \max_{A \subseteq A_n} \{ \sum_{i' \in A_1} (p_{1,i'} - c_{1,i'} - \lambda) v_{1,i'} - \lambda \sum_{i'' \in A_2} v_{2,i''} \} \geq \lambda,$$

since the constraint is equivalent to

$$\max_{A \subseteq A_n} \{ \sum_{i' \in A_1} (p_{1,i'} - c_{1,i'} - \lambda) v_{1,i'} \} \geq \lambda \left(1 + \sum_{i'' \in A_2} v_{2,i''}\right)$$

Can be solved by ranking $p_{ni} - c_{ni}$

Results on assortment-only competition

- **Best response (no display constraint):**
  (i) Retailer n's best response profit decreases in the net value competing firm's product line. (very intuitive)
  (ii) In the best responses, if a firm increases her product line's value, the competing firm would do so.

- **Equilibrium existence (no display constraint):**
  There always exists an equilibrium in the assortment-only competition.

- **Best equilibrium (no display constraint):**
  When there are multiple equilibria, one (the smallest) Pareto-dominates all the others (according to Proposition 1.(i)).

- *No pure-strategy equilibrium with display constraints*

Results on joint assortment and pricing competition

- Assumed
  - $p_{ni} \geq c_{ni} + r_{ni}$ (marginal cost + some mark-up)
  - Simultaneously selected assortment and prices

- The optimal prices are

$$p_{ni}^{*} = c_{ni} + \max\left\{\frac{1}{\alpha_{ni}} + \lambda_n, r_{ni}\right\}$$

- **Equilibrium existence and Pareto-dominance:**
  There always exists a pure-strategy Nash equilibrium. When there are multiple equilibria, one dominates all the others.

- **Capacity limit:**
  In equilibrium, firms always use their full capacity.

Conclusions

- The authors studied a joint assortment and pricing competition under the MNL model.
- When the firms' product line has no overlap:
  - There always exists one equilibrium that Pareto-dominates all the others.
  - Competition leads to a broader set of products.
- (Although omitted) It included some analytical results when there exist common products to the firms.

Thank you for your listening.

Do you have any question?