The Value of Location in Keyword Auctions

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The value of location in keyword auctions

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More clicks (=Revenue) but it costs more

Less clicks (=Revenue) but it costs less
Which slot is better if we compare cost and revenue?

More clicks (=Revenue) but it costs more

Less clicks (=Revenue) but it costs less
### Fundamentals of keyword auctions

The generalized second price mechanism

<table>
<thead>
<tr>
<th>Slot Position</th>
<th>Bid of Advertiser</th>
<th>Price for Slot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slot 1</td>
<td>1ST Bid</td>
<td>P = 2ND Bid</td>
</tr>
<tr>
<td>Slot 2</td>
<td>2ND Bid</td>
<td>P = 3RD Bid</td>
</tr>
<tr>
<td>Slot 3</td>
<td>3RD Bid</td>
<td>P = 4TH Bid</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slot k-1</td>
<td>i-1 Th Bid</td>
<td>P = i Th Bid</td>
</tr>
<tr>
<td>Slot k</td>
<td>i Th Bid</td>
<td>P = i+1 Th Bid</td>
</tr>
<tr>
<td>Slot k+1</td>
<td>i+1 Th Bid</td>
<td>P = i+2 Th Bid</td>
</tr>
<tr>
<td>Slot k+2</td>
<td>i+2 Th Bid</td>
<td>P = i+3 Th Bid</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Truthful Bidding & Balanced Bidding

Truthful Bidding

• An auction mechanism is said to be truthful if the dominant strategy for each bidder is to submit a bid equal to its valuation.
• The famous example of a truthful mechanism is the Vickrey-Clarke-Groves (VCG) auction.
Truthful Bidding & Balanced Bidding

Balanced Bidding

• An auction mechanism is said to be truthful if the dominant strategy for each bidder is to submit a bid equal to its valuation.
Valuations & Profit

In keywords auction, each advertiser has different value on each slot, so author used 4 probability distributions to draw valuations:

- Uniform Distribution
- Normal Distribution
- Exponential Distribution
- Pareto
Valuations & Profit

Advertisers’ distribution of valuation (4 models)

• Uniform model

\[
F_V(x) = \begin{cases} 
0 & \text{if } x < a \\
\frac{x-a}{b-a} & \text{if } a \leq x < b \\
1 & \text{if } x \geq b.
\end{cases}
\]

- \(a\) = lower bound, \(b\) = upper bound

• Normal model

\[
F_V(x) = \int_{-\infty}^{x} \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(y-\mu)^2}{2\sigma^2} \right] dy
\]

- \(\mu\) = mean value,
- \(\sigma\) = standard deviation
Valuations & Profit

Advertisers’ distribution of valuation (4 models)

- **Exponential model**

  \[ F_V(x) = 1 - e^{-\lambda(x-a)}, \quad x \geq a \]

  - \( a \) = lower bound, \( \lambda \) = scale factor

- **Pareto model**

  \[ F_V(x) = 1 - \left( \frac{a}{x} \right)^\beta, \quad x > a \]

  - \( a \) = minimum valuation & scale factor, \( \beta \) = shape parameter
Valuations & Profit

Zipf parameter

- In Zipf model, probability that the user clicks on the slot $j$ is:

$$\theta_j \propto \frac{1}{j^\alpha}, \quad \text{where } \alpha \in \mathbb{R}^+ \text{ is the Zipf parameter}$$

- $\theta_j = \text{Probability that user clicks on slot } j$

$$\sum_{j=1}^{S} \theta_j = 1$$

- $S = \text{total numbers of slots}$
Valuations & Profit

Advertisers’ profit

• Expected profit of ith advertiser receiving kth slot:

\[ U_i(k) = \theta_k \left( v_i - b_{(k+1)} \right) \]

- \( \theta_k \) = Probability that user clicks on slot k , \( v \) = value of advertiser
- \( b \) = bid of advertiser

• If we introduce the Zipf model to the model above

\[ U_i(\alpha, k) \propto \frac{1}{k^\alpha} \left[ v_i - b_{(k+1)} \right], \quad \alpha \in \mathbb{R}^+ \]
Expected Profit

Expected profit $U_i(\alpha, k)$ for the bidder submitting the $i$th highest bid when obtaining the $k$th slot

$$U_i(\alpha, k) \propto \frac{1}{k^\alpha} \mathbb{E}[V_i - V_{(k+1)}] = \frac{1}{k^\alpha} \left( \mathbb{E}[V_i] - \mathbb{E}[V_{(k+1)}] \right)$$
Expected Profit

Uniform model

- The expected value of the ordered valuation:
  \[ E[V_{(r)}] = a + (b - a) \frac{A + 1 - r}{A + 1} \]
  - \( a \) = lower bound, \( b \) = upper bound

- The general expression of the expected profit:
  \[ U_i(\alpha, k) \propto \frac{1}{k^\alpha} \left[ \frac{A + 1 - i}{A + 1} - \frac{A + 1 - (k + 1)}{A + 1} \right] \propto \frac{k + 1 - i}{k^\alpha} \]

- The location of the optimal slot:
  \[ k_{\text{max}} = \arg\max_k U_i(\alpha, k) \]
Expected Profit

Exponential model

• The expected value of the ordered valuation:

\[ \mathbb{E}[V_{(r)}] = a + \frac{1}{\lambda} \sum_{j=r}^{A} \frac{1}{j} \]

- \( a \) = lower bound, \( \lambda \) = scale factor

• The general expression of the expected profit:

\[ U_i(\alpha, k) \propto \frac{1}{k^\alpha} \sum_{j=i}^{k} \frac{1}{j} \]
Expected Profit

Pareto model

- The expected value of the ordered valuation:

\[ \mathbb{E}[V_{(r)}] = a \frac{A!}{(r-1)!} \frac{\Gamma(r - \frac{1}{\beta})}{\Gamma(A + 1 - \frac{1}{\beta})} \]

- \( a \) = minimum valuation & scale factor, \( \beta \) = shape parameter

- The general expression of the expected profit:

\[ U_i(\alpha, k) \propto \frac{1}{k^\alpha} \frac{\Gamma(i - 1/\beta)}{(i - 1)!\Gamma(A + 1 - 1/\beta)} - \frac{1}{k^\alpha} \frac{\Gamma(k + 1 - 1/\beta)}{k!} \]
Optimal Slot Under Truthful Bidding

Simulation Result: Optimal Slot Under

Truthful Bidding
- Uniform Distn
- Normal Distn
- Exponential Distn
- Pareto

Balanced Bidding
- Uniform Distn
- Normal Distn
- Exponential Distn
- Pareto
As long as Zipf parameter is lower than 1, the lowest-positioned slot results to be the profit maximizing slot for all the bidders.

With Zipf parameter larger than 1, the preferences distribute more evenly, but the lowest-position slot is still preferred one.
We note a remarkably similar behavior in the case of the uniform distribution.

Profit profiles for bidders submitting the largest bid reveal that top-positioned slot is preferred one for most Zipf parameter.

Under the normal distribution for click valuations.
Optimal Slot Under Truthful Bidding

Under the exponential distribution for click valuations

- For all but the advertiser with the largest valuation, optimal slot is not that assigned by bid ranking
- And again, the phenomenon is more marked as Zipf parameter gets lower
• We see here a more graceful movement towards the lowest-positioned slot, but the conclusions drawn from other models are confirmed.
Optimal Slot Under Truthful Bidding

Simulation Result: Optimal Slot Under

Truthful Bidding
- Uniform Distn
- Normal Distn
- Exponential Distn
- Pareto

Balanced Bidding
- Uniform Distn
- Normal Distn
- Exponential Distn
- Pareto
For the lowest value of Zipf parameter six out of ten winning bidders have the lowest-positioned slot as their favorite.

When $\alpha<1$ optimal slot location become progressively lower ($\alpha = \text{Zipf parameter}$).
Optimal Slot Under Balanced Bidding

Under the normal distribution for click valuations

- We see a less steep movement of the optimal slot location than in the truthful bidding case.
- For bidder with the largest bid we see instead that profit's growth is more marked than in the truthful bidding case.
Optimal Slot Under Balanced Bidding

Under the exponential distribution for click valuations

- The optimal slot location is shown in left figure above where the preference for the lowest-positioned slots is confirmed once more
• Lower values of Zipf parameter lead to a faster movement towards the lowest-positioned slots
• For bidder submitting the largest bid, unlikely in uniform, normal or exponential model, top-positioned slot is the most preferred one
Conclusions

• A slot lower than the assigned one results to be the optimal one that maximizing the expected profit of the advertiser.
• The bottom-positioned slot appears to be the most preferred one, thanks to its low price that compensates for the lower proportion of clicks.
• The overall consequence is that in most cases advertisers pay more for something that is not the optimal solution for them.

\[ \begin{align*}
\text{top-positioned slot} & < > \text{Slot 1} \\
& < > \text{Slot 2} \\
& < > \text{Slot N} \\
\text{bottom-positioned slot} &< > \text{Optimal.. Preferred..}
\end{align*} \]
Any Questions?